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The nuclear symmetry energy

A.E.L. Dieperink[1], **D. van Neck**[2]

[1] Kernfysisch Versneller Instituut, NL-9474AA Groningen, The Netherlands
[2] Laboratory of Theoretical Physics, Gent University, Gent, Belgium

E-mail: dieperink@kvi.nl

Abstract. The role of isospin asymmetry in nuclei and neutron stars is discussed, with an emphasis on the density dependence of the nuclear symmetry energy. Results obtained with the self-consistent Green function method are presented and compared with various other theoretical predictions. Implications for the equation of state of a neutron star are discussed, and also possible constraints obtained from finite nuclei.

1. Introduction

The Equation of state (EoS) of a neutron star depends strongly on the nuclear symmetry energy as a function of density. Various many-body calculations using a realistic NN interaction as input (BHF [1] or Dirac-BHF[2] and the variational method [3]) lead to rather different results for the symmetry energy. This can be ascribed in part to differences in the treatment of correlations, in particular isovector tensor correlations, which come from pion and rho-meson exchange. For example, in a mean-field (Hartree) approach the pion does not contribute, while on the other hand in an effective field theory (EFT) type of approach usually only the pion is treated explicitly. To get an idea about the importance of correlations I briefly discuss the "self-consistent Green function method" and compare the obtained result with those of BHF.
In view of the rather large differences between the various calculations of the symmetry energy present also at subsaturation densities naturally the question arises whether one can obtain empirical constraints from finite nuclei. To this end one needs to decompose the symmetry energy into the bulk and surface terms. Implications for the EoS of a neutron star and the proton fraction are briefly mentioned.

2. The EoS and symmetry energy in nuclear matter

The symmetry energy is defined by a Taylor series expansion of the energy density in terms of the asymmetry $\alpha = (N-Z)/A$, namely, $E(\rho,\alpha) = E(\rho,0) + S(\rho)\alpha^2 + O(\alpha^4) + ...$
By expanding around saturation density ρ_s, the symmetry energy can be expressed as
$S(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial\alpha^2}|_{\alpha=0} = a_v + \frac{p_0}{\rho_s^2}(\rho-\rho_s) + \frac{\Delta K}{18\rho_s^2}(\rho-\rho_s)^2 + ...$
The empirical value of the bulk symmetry energy is $a_v = 29 \pm 2$ MeV; p_0 is not well known, and calculations cover a large range, from ~ 2(non-relativistic) to 6 (RMF) MeV.fm^3. The curvature ΔK plays role in the analysis of the isoscalar giant monopole resonance (ISGMR), and is found to be negative in a recent experiment on the Sn isotopes [4].
As a function of density, ρ, and proton fraction, $x = (1-\alpha)/2$, the pressure in uniform matter is [6] $P(\rho,x) = \rho^2\frac{\partial E(\rho,x)}{\partial\rho} = \rho^2[E'(\rho,\frac{1}{2}) + S'(\rho)(1-2x)^2 + ..]$.

Using $\beta-$equilibrium, $\mu_e = \mu_n - \mu_p = -\frac{\partial E(\rho,x)}{\partial x}$, and $\mu_e = [3\pi^2\rho x]^{1/3}$ the proton fraction turns out to be quite small, $x_s \sim \frac{1}{\rho_s}[4S(\rho_s)/\hbar c]^3 \sim 0.04$, and hence the pressure is to a large extent determined by the derivative of the symmetry energy: $P(\rho_s) = \rho_s(1-2x_s)[\rho_s S'(\rho_s)(1-2x_s) + S(\rho_s)x_s] \sim p_0$.

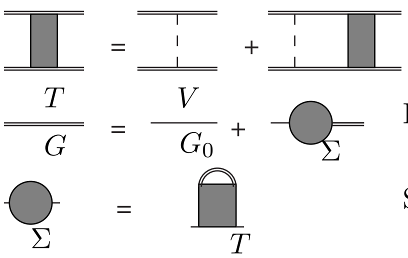
2.1. Many-body methods

A calculation of the bulk symmetry energy basically amounts to taking the difference of the energy density of SNM and PNM. At present for realistic interactions the nuclear matter many-body system cannot be solved without making approximations. One can distinguish two main categories of approaches.

- Methods using nucleonic degrees of freedom starting from input NN phase shifts.
  In this category the most commonly used approaches are the variational approach using correlated basis states [3] and the (lowest-order) Brueckner-HF method. The latter is simple to apply but has various short-comings: it is basically a mean field approach, it treats particles and holes on different footing, and does not saturate at the empirical density. Also with the same NN force the results of the BHF and variational do not agree. Recently a generalization of the BHF has been developed and applied by several groups [9, 10, 11], the self-consistent Green function or in-medium T-matrix method. To illustrate its potential below a comparison of the symmetry energy between the SCGF and BHF methods is presented in which (tensor) correlations play an important role.
  In practice the relativistic Dirac BHF [2] appears to do better in describing saturation properties (in SNM) than the BHF, but for the rest has similar theoretical deficiencies.
- Effective approaches using parameters fitted to many-body data. Non-relativistic (using Skyrme interactions) or relativistic mean field methods (quite successful for symmetric nuclear matter, but less in isospin asymmetric cases; I refer to the talk of Ring at this meeting). The more recent approaches based upon chiral effective field theory (EFT) are interesting (but not yet trustworthy at high density) [12].

2.2. SCGF approach

The Brueckner-HF method, which allows one to use singular realistic interactions, is essentially a mean field approach with a sharp Fermi surface defined by k_F, and has several shortcomings: it does not saturate at the empirical density and tends to overbind SNM. In the self-consistent Green function (SCGF) method (also referred to as in-medium T-matrix approach) the depletion of the fermi surface due to short-range correlations is treated explicitly. I briefly summarize the formalism. The three defining equations are:



ladder summation: $T = V + VGGT$

Dyson equation: $G = G_0 + G_0\Sigma G$

Self-energy: $\Sigma(k,\omega) = \mathrm{Re}\Sigma + i\mathrm{Im}\Sigma = iTr[GT]$

As a result one has to solve three coupled equations. Let me summarize the main properties:

- In SCGF holes and particles treated on equal footing in contrast to BHF,
- It constitutes a "conserving approximation", which means that the self-energy is defined in terms of a functional Φ, $\Sigma(k) = \partial\Phi/\partial G(k)$, hence $\epsilon_F = E_B/A$ (Hugenholz-van Hove theorem, which is violated in BHF),

- The SCGF equations can be solved by iteration and discretization [10],
- A complication is the occurrence of "pairing instabilities" in the hole-hole channel for low temperatures $T < T_c$. This can be cured by introducing "anomalous propagators" like $< aa >$, $< a^\dagger a^\dagger >$ etc.
- The binding energy can be obtained in terms of the spectral function

$$\frac{E}{A} = \frac{1}{2\rho} \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\varepsilon_F} d\omega \left(\frac{k^2}{2m} + \omega \right) S_h(k,\omega), \tag{1}$$

where the spectral function $S_h = \frac{1}{\pi} Im G(k,\omega) = \frac{Im\Sigma(k,\omega)}{(\omega-\epsilon-Re\Sigma)^2+Im\Sigma^2}$. Whereas in BHF only the occupied states contribute explicitly, $S_h(k,\omega) \to \theta(k_f - k)\delta(\omega - \epsilon_k)$, in SCGF 50% of the contribution comes from $k > k_F$; this leads to a doubling of E_{kin} and E_{pot}, and a smaller $E(\rho)$ (increasing with density).

In Fig. 1 the results of BHF (in the continuum choice option) and SCGF are compared for SNM and PNM.

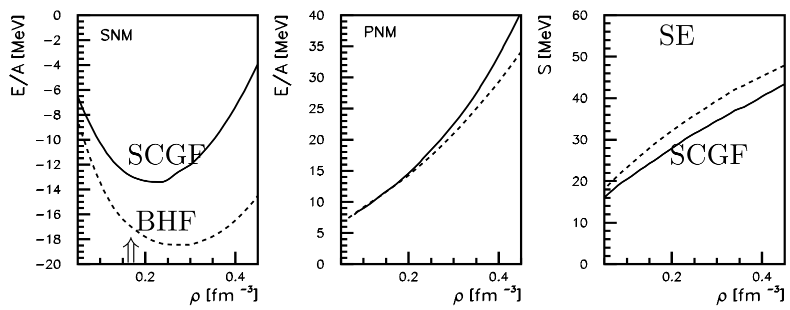


Fig. 1 SNM(left), PNM (middle) and symmetry energy (right) as a function of density calculated with Reid93; full line SCGF, dashed line: BHF [13].

Comparing the SCGF with BHF results one sees for SNM ln the SCGF gives less binding and a shift of the saturation density towards lower densities; for PNM the effect is much smaller, due to absence of T=0 tensor force. As a result the symmetry energy is reduced by about 2 MeV compared to BHF.

2.3. Density dependence of S: Comparison

In Fig. 2 we compare a representative set of results for the density dependence of the symmetry energy from various approaches. One sees that differences occur not only for larger densities but also for $\rho < \rho_s$, relevant for finite nuclei. The SCGF result [13] is close to the variational result [3]. Relativistic approaches yield a larger slope than non-relativistic ones.

It should be noted that effects from three-body forces (3BF) (in non-relativistic methods) have not been included yet in SCGF. If one assumes that the result obtained for 3BF with BHF [14] has a general validity one expects that inclusion of the 3BF in SCGF leads to a much stiffer energy density and will bring all non-relativistic results at higher densities much closer to the covariant ones (see Fig. 2).

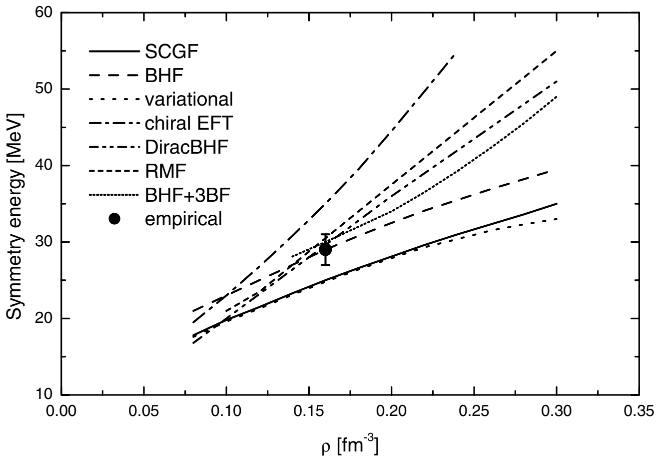


Fig.2 Symmetry energy vs density ρ in various approaches discussed in the text.
Depending on the parametrization the RMF results actually form a broad band, with on the average a steeper slope than BHF.

3. Possible constraints from finite nuclei

Recently Furnstahl [7] pointed out that in mean field approaches a correlation exists between the density dependence of the symmetry energy in nuclear matter and neutron skin in finite nuclei. At first sight this appears surprising since it involves a relation between a surface property and a bulk property, and therefore it is of interest to try to understand the physical origin of this correlation. This can be achieved in several ways. A simple approach is based upon a generalization of the liquid drop model [8]. In practice one first decomposes the symmetry energy in nuclei into bulk and surface contributions; the latter, which can be related to the neutron skin, can be used to constrain the nuclear matter symmetry energy at subsaturation density. We follow a recent discussion by Danielewicz [8] (see also [5]).

- The conventional Bethe-von Weizsäcker (liquid drop) formula expresses the binding energy of a nucleus with N neutrons and Z protons as a sum of bulk, surface, symmetry and Coulomb energies $E_A = -E_B A + E_s A^{2/3} + S_v A(1-2Z/A)^2 + E_C \frac{Z^2}{A^{1/3}}$. It does not distinguish between surface (s) and volume (v) symmetry energies. This can be achieved by partitioning the particle asymmetry as $N - Z = N_s - Z_s + N_v - Z_v$. The total symmetry energy S^A takes on the form $S^A = S_v \frac{(N_v - Z_v)^2}{A} + S_s (N_s - Z_s)^2 / A^{2/3}$. Minimizing under fixed $N - Z$ leads to an improved BW formula

$$E_A = -E_B A + E_s A^{2/3} + S_v A \frac{(1-2Z/A)^2}{1 + S_v A^{-1/3}/S_s} + E_C \frac{Z^2}{A^{1/3}}. \quad (2)$$

- The same approach yields also a relation between the neutron skin ΔR, and S_s, S_v, namely $\frac{R_n - R_p}{R} = \frac{A(N_S - Z_S)}{6NZ} = \frac{A}{6NZ} \frac{N - Z - a_c Z A^{2/3}}{1 + S_s A^{1/3}/S_v}$. Here the Coulomb contribution is essential; e.g. for $N = Z$ ΔR is negative due to the Coulomb repulsion of the protons.

4. Results for S and neutron skin

Using eq.(2) we have fitted the quantities S_v, S_s to a nuclear ground state energies [15] correcting for shell effects. The results depend on details of how the fit is performed, e.g. a replacement of $(N-Z)^2$ by $4T(T+1)$ reduces the best fit value of S_s by 30%. This and other details are presently under investigation [17]. We note that the neutron skin is correlated with the ratio S_v/S_s and could be used as a constraint. Examples of nuclei studied are the Sn isotopes and ^{208}Pb. However, at present the precise extraction of the neutron radius from various experiments appears rather model dependent. If the value for ^{208}Pb from antiprotonic atoms [16] (ΔR =0.13 fm) is included in the fit a smaller value of $S_v/S_s = 1.5$ is obtained.

Table 1. result for symmetry energies

	present	ref [8]	ref [5]
S_v	28	29	27.3
S_v/S_s	1.3	1.7	1.68

To relate the ratio S_v/S_s to the density dependence of S in nuclear matter below subsaturation density, ρ_1, one may use the local density approximation and integrate across the surface [8] $S_v/S_s \approx \frac{3}{R\rho_s} \int dr \frac{\rho(r)}{\rho_s} (\frac{S(\rho_s)}{S(\rho)} - 1)$. (Note: if S is independent of ρ then $S_s \to \infty$.)
Taking $\rho_1 = \frac{2}{3}\rho_s$ leads to $s = S(\rho_1)/S(\rho_s) = 0.7 \pm 0.1$ in nuclear matter. This (preliminary) value is roughly in between the predicted value of nonrelativistic models, $s \sim 0.8$, and the strong density dependence found in covariant approaches, $s \sim 0.6$.

5. Constraints from neutron stars

Given the energy density the Tolman-Oppenheimer-Volkov equation leads to mass vs radius, maximum mass, sensitive to pressure at intermediate densities p_o [6]. If in the future one succeeds in measuring the mass and radius of the *same* star one will be able to discriminate between various EoS. At present there exist only indirect indications. For brevity I only mention the cooling process.
The most efficient cooling process is the direct Urca process $n \to p + e^- + \bar{\nu}, \quad p \to n + e^+ + \nu$. This process is only allowed if energy and momentum are conserved, i.e., if the condition $k_{Fe} + k_{Fp} > k_{Fn}$ is fulfilled, which means that the proton fraction $x > 1/9$. In beta-equilibrium x is determined by the symmetry energy: $x \sim 0.05[S(\rho)/S(\rho_s)]^3 \rho_s/\rho(1-2x)^3$. Since the proton superfluidity and pairing gap depend strongly on the proton density and thus on x this is another manifestation of the role of the symmetry energy in neutron star properties. Since there are no strong indications that fast cooling occurs EoS's with too steep a slope are likely inconsistent.

6. Conclusions

The symmetry energy is the central quantity in calculating neutron star properties. Theoretical results for the density dependence vary strongly, since they treat correlations and in particular the tensor force in different ways. Constraints on subsaturation density from surface symmetry energies and skins from nuclei are useful but require a careful analysis.

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