ChemPubSoc Europe
DOI: 10.1002/cphc.201600433
CHEMPHYSCHEM
Articles

Atom and Bond Fukui Functions and Matrices: A Hirshfeld-I Atoms-in-Molecule Approach

Ofelia B. Oña,[a] Olivier De Clercq,[b] Diego R. Alcoba,[c] Alicia Torre,[d] Luis Lain,[d] Dimitri Van Neck,[e] and Patrick Bultinck*[b]

The Fukui function is often used in its atom-condensed form by isolating it from the molecular Fukui function using a chosen weight function for the atom in the molecule. Recently, Fukui functions and matrices for both atoms and bonds separately were introduced for semiempirical and ab initio levels of theory using Hückel and Mulliken atoms-in-molecule models. In this work, a double partitioning method of the Fukui matrix is proposed within the Hirshfeld-I atoms-in-molecule framework. Diagonalizing the resulting atomic and bond matrices gives eigenvalues and eigenvectors (Fukui orbitals) describing the reactivity of atoms and bonds. The Fukui function is the diagonal element of the Fukui matrix and may be resolved in atom and bond contributions. The extra information contained in the atom and bond resolution of the Fukui matrices and functions is highlighted. The effect of the choice of weight function arising from the Hirshfeld-I approach to obtain atom- and bond-condensed Fukui functions is studied. A comparison of the results with those generated by using the Mulliken atoms-in-molecule approach shows low correlation between the two partitioning schemes.

1. Introduction

Chemistry is basically the study of reactions. Hence, understanding how different compounds react to form new products is of crucial importance, and there is a continuing quest to develop methods and concepts that would allow chemists to know a priori if and how molecules will react. Of course, this quest has been ongoing since the start of what would eventually become known as chemistry and, based on empirical evidence, an entire toolbox of concepts and quantities was developed to yield such information. Relatively recently, quantum chemical theories have been used for the prediction of molecular reactivity. One of the more well-known theories in this regard is Density Functional Theory (DFT)[1] where so-called conceptual or chemical DFT[1,2] tries to unite several of the above-mentioned, often semi-classical, chemical concepts and theories in one unifying theoretical model. One of the key tools used by many chemists to understand molecular reactivity is frontier molecular orbital (FMO) theory,[3,4] which describes reactivity by using the highest occupied molecular orbital (HOMO) and the lowest unoccupied molecular orbital (LUMO) of a molecule. However, it is also well known that using merely the HOMO and the LUMO does not always suffice to predict or understand reactivity. In conceptual DFT, FMO theory is an approximation to the so-called Fukui function. The Fukui function corresponds to the first mixed second-order derivative of the energy [Eq. (1)]:[5–7]

$$f(\mathbf{r}) = \left(\frac{\partial \left(\frac{\delta E}{\delta v(\mathbf{r})} \right)_N}{\partial N} \right)_{v(\mathbf{r})} = \left(\frac{\partial \rho(\mathbf{r})}{\partial N} \right)_{v(\mathbf{r})} \tag{1}$$

where E is the energy, N is the total number of electrons in the system, and $v(\mathbf{r})$ is the external potential. $\rho(\mathbf{r})$ is the electron density function that in the non-degenerate case corresponds to the electron density of the molecule. In the case of degenerate states, the density to be used is derived from degenerate perturbation theory.[8–10] The Fukui function indicates regions in a molecule that exhibit higher reactivity with respect to reac-

---

[a] Dr. O. B. Oña
Instituto de Investigaciones Fisicoquímicas Teóricas y Aplicadas
Universidad Nacional de la Plata, CCT La Plata
Consejo Nacional de Investigaciones Científicas y Técnicas
Diag. 113 y 64 (s/n), Sucursal 4, CC 16
1900 La Plata (Argentina)

[b] O. De Clercq, Prof. Dr. P. Bultinck
Department of Inorganic and Physical Chemistry
Ghent University, Krijgslaan 281 (S3)
9000 Gent (Belgium)
E-mail: Patrick.Bultinck@UGent.be

[c] Dr. D. R. Alcoba
Departamento de Física, Facultad de Ciencias Exactas y Naturales
Universidad de Buenos Aires
Ciudad Universitaria, 1428 Buenos Aires (Argentina)
Instituto de Física de Buenos Aires
Consejo Nacional de Investigaciones Científicas y Técnicas
Ciudad Universitaria, 1428 Buenos Aires (Argentina)

[d] Prof. Dr. A. Torre, Prof. Dr. L. Lain
Departamento de Química Física
Facultad de Ciencia y Tecnología
Universidad del País Vasco
Apdo. 644 E-48080 Bilbao (Spain)

[e] Prof. Dr. D. Van Neck
Center for Molecular Modeling
Ghent University, Technologiepark 903
B-9052 Zwijnaarde (Belgium)

Supporting Information and the ORCID identification number(s) for the author(s) of this article can be found under http://dx.doi.org/10.1002/cphc.201600433.

tions that involve a change in the number of electrons in the molecule. The Fukui function is not only of interest by itself but is also an important function for generating many other conceptual (DFT) quantities.[1,2] Equation (1) shows that the Fukui function is an **r**-dependent function that describes the change in the electron density at the point **r** upon a change in the number of electrons in the molecule, under the constraint that the external potential remains unaltered. The electron density has slope discontinuities at the integer number of electrons and, consequently, the left-side and right-side derivatives are different. Hence, one needs to consider two Fukui functions. The Fukui function for an increase in the number of electrons will be denoted $f^+(\mathbf{r})$ and for a decrease as $f^-(\mathbf{r})$. They are expressed as Equations (2) and (3):[1,2]

$$f^+(\mathbf{r}) = \left(\frac{\partial\rho(\mathbf{r})}{\partial N}\right)^+_{v(\mathbf{r})} = \lim_{\varepsilon\to 0^+}\left(\frac{\rho^{N+\varepsilon}(\mathbf{r}) - \rho^N(\mathbf{r})}{\varepsilon}\right) \tag{2}$$

$$f^-(\mathbf{r}) = \left(\frac{\partial\rho(\mathbf{r})}{\partial N}\right)^-_{v(\mathbf{r})} = \lim_{\varepsilon\to 0^+}\left(\frac{\rho^{N}(\mathbf{r}) - \rho^{N-\varepsilon}(\mathbf{r})}{\varepsilon}\right) \tag{3}$$

in which $\rho^{N+\varepsilon}(\mathbf{r})$ is the electron density of a molecule at point **r** for the molecule with $N+\varepsilon$ electrons and similarly for the $\rho^{N-\varepsilon}(\mathbf{r})$ density function (molecule with $N-\varepsilon$ electrons). Usually, the calculation of both limits is performed by means of the finite difference method with $\varepsilon=1$; it provides the determination of the Fukui functions through a difference of electron densities for the molecule and the ionic species keeping the geometry frozen. The formulation $\varepsilon=1$ leads to an exact result[11–13] for an exact calculation so that it is not a crude approximation. An electron that is accepted in a molecule tends to be placed in the regions where $f^+(\mathbf{r})$ is large owing to the ability of the molecule to stabilize additional electrons at exactly these locations, similarly in the case of the loss of an electron in a molecule, the electron hollow tends to be situated at places where $f^-(\mathbf{r})$ is large.

Recently, Bultinck et al.[14,15] and later Alcoba et al.[16–18] extended the Fukui function to a Fukui matrix in the following way [Eq. (4)]:

$$f(\mathbf{r},\mathbf{r}') = \left(\frac{\partial\rho(\mathbf{r},\mathbf{r}')}{\partial N}\right)_{v(\mathbf{r})} \tag{4}$$

where $\rho(\mathbf{r}, \mathbf{r}')$ is the first-order reduced density matrix. This Fukui matrix $f(\mathbf{r}, \mathbf{r}')$ has previously been shown to lead to rich new insights into the properties of the Fukui function and allows rationalization of the quality of a frozen molecular orbital or Koopmans approximation to it.[14,15] The Fukui function corresponds to the diagonal of the Fukui matrix in the same way as the density function corresponds to the diagonal of the first-order reduced density matrix.

In practice, the Fukui function $f(\mathbf{r})$ can be used to locate reactive sites in a molecule or as a field in, for example, 3D QSAR.[19,20] Yet, there is a large preference among many users of conceptual DFT to use condensed Fukui functions instead as such a coarse grain representation is easier to handle than a 3D field. These are obtained through a combination of parameter differentiation (with respect to N) and integration over the domain of an atom in the molecule (AIM). When introducing these for the first time, Yang and Mortier[21] relied on the commutability of parameter differentiation (finite-difference here) and integration over the AIM to find that the atom-condensed Fukui function f_A for an atom A is given by Equations (5) and (6):

$$f_A^+ = q_A(N+1) - q_A(N) \tag{5}$$

$$f_A^- = q_A(N) - q_A(N-1) \tag{6}$$

where $q_A(N+1)$, $q_A(N-1)$, and $q_A(N)$ are the charges on atom A in the molecule with $N+1$, $N-1$, and N electrons, respectively. It is important to stress that these authors used Mulliken charges for which the above-mentioned commutability holds. This is not true in general, as discussed in detail previously by Bultinck et al.[22]

In general, the atom-condensed Fukui function can be computed either as Equation (7) or (8):

$$f_A^+ = \int w_A(\mathbf{r})\left(\frac{\partial\rho(\mathbf{r})}{\partial N}\right)^+_{v(\mathbf{r})} d\mathbf{r} \tag{7}$$

$$f_A^+ = \int \left(\frac{\partial(w_A(\mathbf{r})\rho(\mathbf{r}))}{\partial N}\right)^+_{v(\mathbf{r})} d\mathbf{r} \tag{8}$$

and the analogous counterparts for electron removal. The $w_A(\mathbf{r})$ are the atomic weight functions, where the separation of the molecule into atoms is performed by using a 3D space partitioning such as Bader's quantum theory of atoms in molecules (QTAIM)[23,24] or overlapping AIM methods. The fact that the expressions above may lead to different results is due to, given a specific AIM method, the possible dependence of $w_A(\mathbf{r})$ on the electronic population of the molecule (N versus $N\pm1$). The two different expressions coincide in the case of the Mulliken method, as used by Yang and Mortier,[21] because the weight is based on a Hilbert space partitioning, which is N-independent (provided the same basis set is used for the neutral and charged molecule). In the original Hirshfeld method,[25] no difference is observed either, owing to the common choice of (the arbitrary) atomic weight functions for both the molecule and molecular ion. In the case of QTAIM and Hirshfeld-I[26] methods, the differences obtained by using both methods may be substantial.[22,27,28] In Equation (7), one considers the entire molecule as undergoing the perturbation first, after which it is partitioned. This approach is called the fragment of molecular response (FMR). Bader has shown, in the context of QTAIM,[24] that Equation (7) is to be preferred based on an atomic variation principle, meaning that one should always first define a molecular property density, which is then integrated over the proper domain. Equation (8), on the other hand, is based on the response of a molecular fragment (RMF) to the perturbation. This equation is much more popular as it is more easily computed through a difference in atomic charges. These quantities are easily available from many quantum chemical programs. In recent work, Morgenstern et al.[29] preferred the second approach [Eq. (8)] in a QTAIM context specifically because it includes the change in the zero flux surface. Several authors have shown that these different approaches produce significantly different values, which even have an important impact on their sign.[27,30] Among these approaches, the FMR one gives a coherent treatment of the hardness kernel. In the remainder of this work, Equation (7) will be used unless otherwise noted.

The purpose of this paper is to introduce a new method to partition Fukui matrices in biatomic parts, as done recently for density matrices by using 3D space AIM partitioning.[31] Once such matrices are available, they can be diagonalized and their spectrum of eigenvalues and the accompanying eigenvectors (called Fukui orbitals) can be studied. The trace of these Fukui matrices gives biatomic condensed Fukui functions based on solely the first-order reduced density matrix, as opposed to cases where the second-order reduced density matrix is used.[32–36] This allows us to attach a degree of reactivity to chemical bonds and atoms separately. The occurrence of negative-atom-condensed Fukui functions is a much debated issue.[27,28,30,37–43] This problem can now be studied in much more detail by examining the eigenvalue distribution of the corresponding Fukui matrices. Finally, we examine to what extent results for condensed Fukui functions depend on the weight function chosen: that is, whether one uses the AIM weights $w_A(\mathbf{r})$ of the neutral molecule or those of the relevant ion. Important requirements to be fulfilled are that the sum of all biatomic Fukui functions must respect normalization to 1, that the sum of all Fukui matrices must be equal to the total molecular Fukui matrix, and that the regular atom-condensed Fukui functions (i.e., where no bonds are considered) must be derivable from the newly introduced atom- and bond-condensed Fukui functions.

2. Theory

The present paper focuses on the use of the Hirshfeld-I AIM,[26] an extension of the Hirshfeld AIM,[25] to obtain weight functions $w_A(\mathbf{r})$. Numerical data using the Mulliken AIM are compared with the Hirshfeld-I results to examine the influence of the AIM method. No derivation is explicitly presented for the Mulliken case as it can be directly derived through repeated application of the Mulliken operator.[44,45] The Hirshfeld-I weight functions are derived from a so-called promolecular density $\rho^0_{\text{promol}}(\mathbf{r},N)$,[25] which is the sum of isolated atomic densities of the atoms in the molecule. The number of electrons assigned to every particular atom can be chosen freely, albeit different from zero. Through an iterative procedure, self-consistent atomic charges (or weights) are found in the following way [Eq. (9)]:

$$w^i_A(\mathbf{r}) = \frac{\rho^0_A\left(\mathbf{r},N^{i-1}_A\right)}{\rho^0_{\text{promol}}(\mathbf{r},N)} = \frac{\rho^0_A\left(\mathbf{r},N^{i-1}_A\right)}{\sum_A \rho^0_A\left(\mathbf{r},N^{i-1}_A\right)} \tag{9}$$

with $N^i_A = \int w^i_A(\mathbf{r})\rho(\mathbf{r})d\mathbf{r}$, $\rho^0_A\left(\mathbf{r},N^i_A\right)$ as the isolated atomic densities, and where i refers to the iteration number. In the first iteration, a regular Hirshfeld calculation is performed by using neutral atomic densities. In the next iterations, the isolated atomic densities correspond to charged atoms (see Bultinck et al.[26] for details). This procedure reduces the dependence of the final weights on the chosen promolecule and in practice always yields the same solution.[46] Beside this major advantage, Hirshfeld-I also exhibits only a minor basis set dependence,[46] and produces good electrostatic potentials on an outer molecular surface from solely monopoles.[47–49] It also offers a good compromise between the conformational dependence of the atomic charges and transferability[48,50] and leads to very good performance in electronegativity equalization models[49,51] compared with the regular Hirshfeld method.[52] In Hirshfeld-I, all atoms extend to infinity and weights vary smoothly, such that they can be projected on a typical basis used in ab initio calculations.

Following the work of Vanfleteren et al.,[31] where a double-index atomic partitioning of the molecular first-order reduced density matrix was proposed, the Fukui matrix is partitioned according to Equation (10):

$$f(\mathbf{r},\mathbf{r}') = \sum_{AB} f_{AB}(\mathbf{r},\mathbf{r}') \tag{10}$$

where

$$f_{AB}(\mathbf{r},\mathbf{r}') = \frac{1}{2}\left(w_A(\mathbf{r})w_B(\mathbf{r}') + w_A(\mathbf{r}')w_B(\mathbf{r})\right)f(\mathbf{r},\mathbf{r}') \tag{11}$$

and the $w_A(\mathbf{r})$ is the atomic weight function, which obeys

$$0 \leq w_A(\mathbf{r}) \leq 1 \text{ and } \sum_A w_A(\mathbf{r}) = 1 \tag{12}$$

Note that, clearly, we always need to distinguish a separate Fukui matrix for electron attachment and removal but, in order not to overload notation, we opt not to include this specification. Moreover, we also distinguish a separate Fukui matrix for electrons with spin α and β as in previous works.[14,16] In the resulting set of $f_{AB}(\mathbf{r},\mathbf{r}')$, we shall distinguish those matrices diagonal in atomic indices (A = B) and off-diagonal ones (A ≠ B), hereafter called atomic and bond Fukui matrices, respectively. These matrices can further be expressed in terms of the molecular orbitals {$i, j, \ldots$} of the neutral molecule,

$$(\mathbf{F}_{AB})_{ij} = < i|f_{AB}(\mathbf{r},\mathbf{r}')|j > \tag{13}$$

The matrix $\mathbf{F}_{AB}$ holding the elements $(\mathbf{F}_{AB})_{ij}$ can be diagonalized, giving rise to Fukui orbitals for each combination AB of two atoms. The eigenvalues in turn express the magnitude of the contributions of these eigenvectors. The trace of the matrix $\mathbf{F}_{AB}$ gives an atom- or bond-condensed Fukui function f_{AB} for each combination of two atoms. The proposed partitioning also fulfills other important requirements including symmetry and the constraints:

$$f_A = \sum_B f_{AB} \tag{14}$$

$$\sum_{A,B} f_{AB} = \sum_A f_A = 1 \tag{15}$$

where f_A corresponds to the traditional atom-condensed Fukui function for an AIM A. It is therefore worth stressing that all these more traditional uses of Fukui functions are completely consistent with the presently suggested procedures. However, the bond Fukui matrices allow a more detailed analysis for a determined chemical reaction as they involve individual pairs of atoms within a molecule and their summation should reproduce the traditional treatment.

When reporting data in tabular form in the Results and Discussion, we take into account that for both the Fukui matrices and condensed Fukui function values we have $\mathbf{F}_{AB}=\mathbf{F}_{BA}$ and $f_{AB}=f_{BA}$. The numerical data consistently report the data for $(2-\delta_{AB})f_{AB}$ and similarly for the Fukui matrix (where the symbol δ_{AB} means the Kronecker delta).

3. Computational Methods

To illustrate the newly introduced biatomic Fukui functions and to shed some light on the choices to be made to compute reactivity indices, B3LYP calculations were performed on the set of ethylene derivatives shown in Figure 1 very recently used by Gonzalez-Suarez et al.[44] and Bultinck et al.[45] using the Hirshfeld-I AIM method (i.e., molecules 1–18 from ref. [44]). The original idea of this set of molecules was to examine the influence of the addition of certain electron-donating and electron-withdrawing groups on the 1–2 bond-condensed Fukui function. Geometry optimizations were carried out by using the Cartesian 6-31G* basis set and all information required for the calculation of the Fukui matrix was extracted from the formatted checkpoint file from Gaussian 03.[53] Hirshfeld-I weight functions were computed by using atomic densities obtained at the same level of theory. All required algebraic manipulations were described previously by Bultinck et al.[14] with straightforward extensions. For all molecules and molecular ions, it is assumed that the number of α spin electrons is always larger than or equal to the number of β spin electrons. In the following, we restrict ourselves to the study of the spin block of the biatomic Fukui matrix with trace 1, that is, the β block for electron removal and the α block for electron addition. The weight functions $w_A(\mathbf{r})$, as the Fukui matrices, are expressed in the basis of the molecular orbitals. Visualization of the resulting Fukui orbitals for each biatomic combination is done by using GaussView5.[54]

4. Results and Discussion

For all molecules, both the electron addition and electron removal Fukui matrices have been computed. In all cases, the normalization of these Fukui matrices has an absolute margin of error less than 10^{-8} for both spins. The biatomic Fukui matrices have been obtained by using Equation (11), and for all the studied systems the sum of the Fukui matrices F_{AB} equals the corresponding total Fukui matrix within an absolute difference of less than 10^{-3} over all elements. The origin of this difference lies in the fact that the requirement $\sum_A w_A(\mathbf{r}) = 1$ is not fulfilled exactly owing to the necessity of numerical integration over a finite grid size to obtain atomic overlap matrices from 3D space AIM methods. In agreement with previous works by Bultinck et al.,[14,15] and Alcoba et al.,[16–18] the total molecular Fukui matrix possesses an interesting eigenvalue spectrum. At the DFT level, the (loosely defined) first-order reduced density matrix is idempotent and consequently the molecular Fukui matrix always has one single eigenvalue exactly equal to 1 and all other eigenvalues are either exactly zero or, as is the case for most eigenvalues, they come in pairs of $+x$ and $-x$ with $|x|<1$, thus pairwise exactly summing zero. Moreover, we again find that in all cases the dominant Fukui orbital (i.e., the eigenvector with unity eigenvalue) has a single

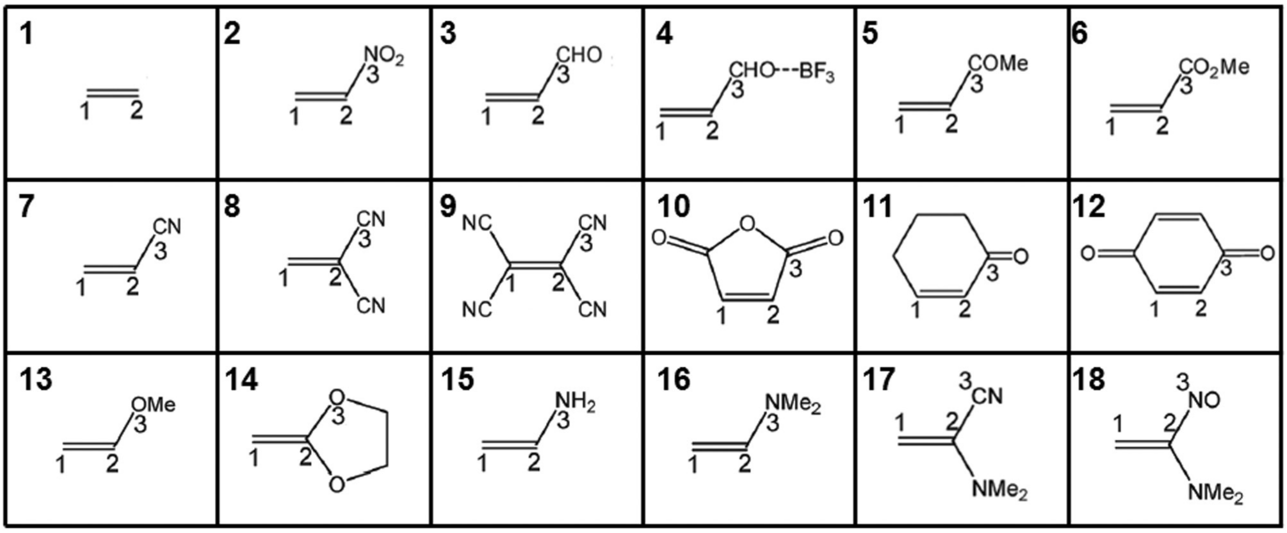

Figure 1. Ethylene derivatives considered in this work with explicit indication of atoms 1, 2, and 3.

coefficient with magnitude of nearly 1 for the molecular orbital that corresponds to the frontier molecular orbital of the neutral compound. These features are also fulfilled in the case of our B3LYP calculations.[14]

To illustrate the information provided by the biatomic Fukui matrices, Table 1 and Table 2 give a detailed overview of the data for C_2H_4 (see Figure 2). We focus on the properties of the obtained data rather than their actual numerical values and not on linking them to the observed reactivity (see ref. [44] for the discussion on the reactivity). These tables clearly show a number of important points. The first one is that for every combination of two atoms, some negative eigenvalues of significant magnitude appear. As expected, the eigenvalue pairing, which manifest for the total Fukui matrix from theories

Table 1. Eigenvalues and traces of $\mathbf{F}^{-}_{AB}$ Fukui matrices for C_2H_4, calculated in the 6-31G* basis set on the B3LYP level of theory by using $w_A(r)$ from the neutral molecule. Eigenvalues are ordered from the lowest value to the highest.

Eigenvalues Fragment	1	2	3	4	5	6	···	32	33	34	35	36	37	38	f^{-}_{AB}
C_1C_1	−0.034	−0.023	−0.009	−0.001	0.000	0.000	···	0.000	0.000	0.000	0.005	0.005	0.007	0.325	0.198
C_1C_2	−0.245	−0.017	−0.013	−0.008	−0.007	−0.002	···	0.001	0.001	0.002	0.006	0.009	0.011	0.405	0.126
C_1H_3	−0.028	−0.005	−0.004	−0.001	−0.001	0.000	···	0.000	0.000	0.000	0.001	0.004	0.005	0.033	0.006
C_1H_4	−0.020	−0.012	−0.004	−0.002	−0.001	0.000	···	0.000	0.000	0.001	0.002	0.008	0.022	0.049	0.057
C_1H_5	−0.020	−0.012	−0.004	−0.002	−0.001	0.000	···	0.000	0.000	0.001	0.002	0.008	0.022	0.049	0.057
C_1H_6	−0.028	−0.005	−0.004	−0.001	−0.001	0.000	···	0.000	0.000	0.000	0.001	0.004	0.005	0.033	0.006
C_2C_2	−0.034	−0.023	−0.009	−0.001	0.000	0.000	···	0.000	0.000	0.000	0.005	0.005	0.007	0.325	0.198
C_2H_3	−0.020	−0.012	−0.004	−0.002	−0.001	0.000	···	0.000	0.000	0.001	0.002	0.008	0.022	0.049	0.057
C_2H_4	−0.028	−0.005	−0.004	−0.001	−0.001	0.000	···	0.000	0.000	0.000	0.001	0.004	0.005	0.033	0.006
C_2H_5	−0.028	−0.005	−0.004	−0.001	−0.001	0.000	···	0.000	0.000	0.000	0.001	0.004	0.005	0.033	0.006
C_2H_6	−0.020	−0.012	−0.004	−0.002	−0.001	0.000	···	0.000	0.000	0.001	0.002	0.008	0.022	0.049	0.057
H_3H_3	−0.006	0.000	0.000	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.000	0.003	0.027	0.053
H_3H_4	−0.003	−0.002	−0.001	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.001	0.002	0.003	0.001
H_3H_5	−0.004	−0.003	−0.002	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.002	0.003	0.004	−0.001
H_3H_6	−0.009	−0.002	−0.001	0.000	0.000	0.000	···	0.000	0.000	0.000	0.001	0.002	0.005	0.008	0.006
H_4H_4	−0.006	0.000	0.000	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.000	0.003	0.027	0.053
H_4H_5	−0.009	−0.002	−0.001	0.000	0.000	0.000	···	0.000	0.000	0.000	0.001	0.002	0.005	0.008	0.006
H_4H_6	−0.004	−0.003	−0.002	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.002	0.003	0.004	−0.001
H_5H_5	−0.006	0.000	0.000	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.000	0.003	0.027	0.053
H_5H_6	−0.003	−0.002	−0.001	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.001	0.002	0.003	0.001
H_6H_6	−0.006	0.000	0.000	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.000	0.003	0.027	0.053
$\sum_{AB} f^{-}_{AB}$															1.000

Table 2. Eigenvalues and traces of $\mathbf{F}^{+}_{AB}$ Fukui matrices for C_2H_4, calculated in the 6-31G* basis set on the B3LYP level of theory using $w_A(r)$ from the neutral molecule. Eigenvalues are ordered from the lowest value to the highest.

Fragment Eigenvalues	1	2	3	4	5	6	···	32	33	34	35	36	37	38	f^{+}_{AB}
C_1C_1	−0.028	−0.023	−0.004	−0.004	−0.001	0.000	···	0.000	0.000	0.001	0.002	0.004	0.005	0.309	0.189
C_1C_2	−0.264	−0.011	−0.009	−0.006	−0.006	−0.005	···	0.002	0.002	0.002	0.005	0.005	0.007	0.352	0.067
C_1H_3	−0.035	−0.005	−0.003	−0.001	0.000	0.000	···	0.000	0.000	0.000	0.001	0.003	0.005	0.044	0.010
C_1H_4	−0.020	−0.013	−0.003	−0.002	0.000	0.000	···	0.000	0.000	0.001	0.001	0.008	0.020	0.069	0.069
C_1H_5	−0.020	−0.013	−0.003	−0.002	0.000	0.000	···	0.000	0.000	0.001	0.001	0.008	0.020	0.069	0.069
C_1H_6	−0.035	−0.005	−0.003	−0.001	0.000	0.000	···	0.000	0.000	0.000	0.001	0.003	0.005	0.044	0.010
C_2C_2	−0.028	−0.023	−0.004	−0.004	−0.001	0.000	···	0.000	0.000	0.001	0.002	0.004	0.005	0.309	0.189
C_2H_3	−0.020	−0.013	−0.003	−0.002	0.000	0.000	···	0.000	0.000	0.001	0.001	0.008	0.020	0.069	0.069
C_2H_4	−0.035	−0.005	−0.003	−0.001	0.000	0.000	···	0.000	0.000	0.000	0.001	0.003	0.005	0.044	0.010
C_2H_5	−0.035	−0.005	−0.003	−0.001	0.000	0.000	···	0.000	0.000	0.000	0.001	0.003	0.005	0.044	0.010
C_2H_6	−0.020	−0.013	−0.003	−0.002	0.000	0.000	···	0.000	0.000	0.001	0.001	0.008	0.020	0.069	0.069
H_3H_3	−0.007	0.000	0.000	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.000	0.005	0.026	0.053
H_3H_4	−0.004	−0.002	−0.001	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.001	0.002	0.006	0.003
H_3H_5	−0.005	−0.004	−0.002	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.002	0.005	0.006	0.001
H_3H_6	−0.007	−0.002	−0.002	0.000	0.000	0.000	···	0.000	0.000	0.000	0.001	0.002	0.007	0.009	0.009
H_4H_4	−0.007	0.000	0.000	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.000	0.005	0.026	0.053
H_4H_5	−0.007	−0.002	−0.002	0.000	0.000	0.000	···	0.000	0.000	0.000	0.001	0.002	0.007	0.009	0.009
H_4H_6	−0.005	−0.004	−0.002	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.002	0.005	0.006	0.001
H_5H_5	−0.007	0.000	0.000	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.000	0.005	0.026	0.053
H_5H_6	−0.004	−0.002	−0.001	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.001	0.002	0.006	0.003
H_6H_6	−0.007	0.000	0.000	0.000	0.000	0.000	···	0.000	0.000	0.000	0.000	0.000	0.005	0.026	0.053
$\sum_{AB} f^{+}_{AB}$															1.000

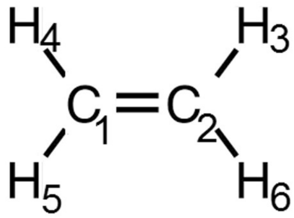

Figure 2. Labels of the C_2H_4 molecule.

giving idempotent density matrices, is not carried through to the biatomic condensed level of the Fukui matrices. The last column in both tables reports the condensed Fukui function for each atomic pair, which is obtained by taking the sum of the traces of the spin blocks of the bond Fukui matrices. These columns thus show that in ethylene at the current level of theory and basis set, the diatomic combination C_1C_2 has the largest condensed Fukui function for the bond Fukui functions. None of the combinations AB gives a positive definite Fukui matrix and, although in the present case most of the bond-condensed Fukui functions result in positive values, these functions can also be negative, as shown in the tables. Results for the other 17 compounds gathered in Figure 1 are reported in the Supporting Information.

By application of Equation (14), Fukui indices condensed to single atoms can be obtained in a straightforward fashion. Table 3 shows the data obtained for C_2H_4. Again, the normalization is respected and the traditionally often used atom-condensed Fukui functions can be recovered easily.

Table 3. Atom-condensed Fukui functions f_A^- and f_A^- using $w_A(r)$ from the neutral molecule.

Atom	f_A^-	f_A^+
C_1	0.325	0.301
C_2	0.325	0.301
H_3	0.088	0.099
H_4	0.088	0.099
H_5	0.088	0.099
H_6	0.088	0.099
$\sum_A f_A$	1.000	1.000

Further analysis of the data shows that, in most of the molecules in the test set, the spin block of the C_1C_2 bond Fukui matrix corresponding to the spin block with trace 1 of the molecular Fukui matrix usually has two dominant eigenvectors with different eigenvalues of opposite sign. This is not the case for the other spin block, which has multiple eigenvectors with much smaller (in absolute value) eigenvalues. These observations are reflected in Figure 3 for the two smaller systems of the set of ethylene derivatives (compounds 1 and 15 of Figure 1).

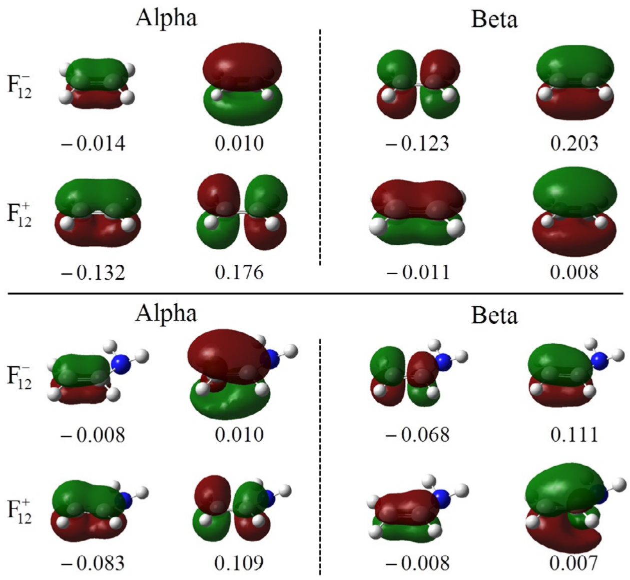

Figure 3. The lowest and highest eigenvalues with their eigenvectors for the two smaller systems of the set of ethylene derivatives (compounds 1 and 15 of Figure 1) using $w_A(r)$ from the neutral molecule, all plotted for a 0.04 isovalue.

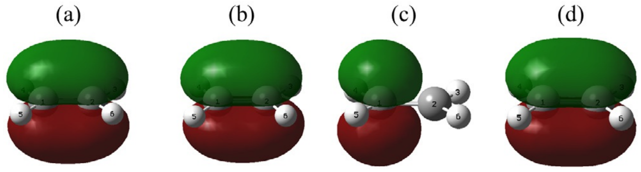

Figure 4. a) The HOMO of C_2H_4 at the B3LYP/6-31G* level of theory, b) the eigenvector with eigenvalue equal to 1 of the total Fukui matrix, and c) and d) the eigenvectors of $\mathbf{F}_{AB}^-$ with the highest eigenvalues of, respectively, the AB = C_1C_1 and C_1C_2 atomic pairs, all plotted for a 0.04 isovalue. These are calculated for the removal of a β electron. The eigenvalues corresponding to the Fukui orbitals are 0.325 and 0.203.

Having established the main characteristics of the Fukui matrices condensed to atoms and atom pairs, we now tackle the analysis of the Fukui orbitals for C_2H_4. Figure 4 shows the HOMO orbital for C_2H_4 along with the Fukui orbitals with the highest eigenvalue of the Fukui matrices for electron removal. The first observation is that these orbitals are well localized around a single atom or bond, as the weighting functions have their highest value close to the atoms A and B. The nature of the test set is such that in the majority of cases, the HOMO and LUMO correspond to π and π* orbitals localized on the C_1C_2 bond. This entails that the cationic species have, compared with the neutral molecule, one electron less in a bonded orbital, and that in the anionic species an electron ended up in the antibonding orbital. The dominant eigenvectors are well localized on the relevant atom pairs. The C_1C_1 dominant eigenvector is clearly very well localized on the C_1 carbon atom. The C_1C_2 dominant eigenvector, on the other hand, has contributions from both C_1 and C_2 as expected. This localization also applies to all other eigenvectors corresponding to the non-negligible eigenvalues of these two-atom combinations. Similar localization is also found for the LUMO orbital and the Fukui matrices for electron addition, as shown in Figure 5.

The choice of the weight function used to calculate atom-condensed Fukui functions f_{AB} should ideally have only a small influence. More precisely, the effect of using either the weight

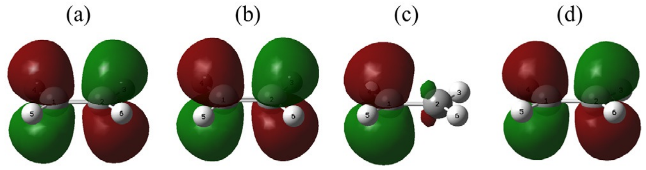

Figure 5. a) The LUMO of C_2H_4 at the B3LYP/6-31G* level of theory, b) eigenvector with the highest eigenvalue equal to 1 of the total Fukui matrix, and c) and d) the eigenvectors of $\mathbf{F}^{+}_{AB}$ with the highest eigenvalues of, respectively, the AB$=C_1C_1$ and C_1C_2 atomic pairs, all plotted for a 0.04 isovalue. These are calculated for the addition of an α electron. The eigenvalues corresponding to the Fukui orbitals are 0.309 and 0.176.

function for the AIM A and B from the neutral molecule or from the molecular ion is hoped to be small. To show the effect of changing the weight functions, a new weight function is used [Eq (16)]:

$$w_A(r) = p \quad w_A^{\text{neutral}}(r) + (1-p) \quad w_A^{\text{ionic}}(r) \tag{16}$$

Here, $w_A^{\text{neutral}}(r)$ and $w_A^{\text{ionic}}(r)$ are the weight functions on the neutral and ionic systems (the cationic or the anionic one for the calculation of the $\mathbf{F}^-$ and $\mathbf{F}^+$ matrices, respectively), and p is a parameter that varies between 0 and 1. Table 4 presents the effect of the variation of p on the f_{12}, f_{23}, and f_1 values by means of the correlation coefficient R^2 between the results by using $w_A(r)$ from the molecular ion and $w_A(r)$ from the neutral molecule. At least for all molecules in the test set there is no strong dependence on the source of the weight functions used in the computation of atom- and bond-condensed Fukui functions. The correlation coefficient R^2 between the values of both types of weights is $>99\%$. There is a somewhat poorer correlation for Fukui matrices for electron addition, but the loss in correlation remains very small.

Table 4. B3LYP 6-31G*, correlation coefficients R^2 between the atom- and bond-condensed Fukui functions for addition or removal of an electron for each molecule using $\{w_A(r)\}$ from the molecular ion to $\{w_A(r)\}$ from the neutral molecule.

p	0.00	0.25	0.50	0.75	1.00
f^-_{12}	0.996	0.998	0.999	1.000	1.000
f^-_{23}	0.998	0.999	1.000	1.000	1.000
f^-_1	0.998	0.999	1.000	1.000	1.000
f^+_{12}	0.988	0.993	0.997	0.999	1.000
f^+_{23}	0.994	0.997	0.999	1.000	1.000
f^+_1	0.998	0.999	0.999	1.000	1.000

Table 5. B3LYP 6-31G* bond Fukui functions between the indicated atoms and the atom-condensed Fukui function on atom 1 for addition or removal of an electron for each molecule using $w_A(r)$ from the neutral molecule. Values in parentheses were calculated by using the Mulliken approach.

System	f^-_{12}	f^+_{12}	f^-_{23}	f^+_{23}	f^-_1	f^+_1
1	0.126 (0.517)	0.067 (−0.607)	0.057 (0.022)	0.069 (−0.056)	0.325 (0.170)	0.301 (0.140)
2	0.040 (0.038)	0.034 (−0.228)	0.008 (0.022)	0.029 (0.212)	0.149 (0.072)	0.200 (0.112)
3	0.030 (0.055)	0.036 (−0.192)	0.020 (−0.127)	0.051 (0.265)	0.134 (0.071)	0.215 (0.127)
4	0.065 (0.270)	0.035 (−0.115)	0.020 (0.039)	0.051 (0.249)	0.178 (0.104)	0.205 (0.127)
5	0.025 (0.052)	0.035 (−0.204)	0.016 (−0.115)	0.041 (0.249)	0.114 (0.058)	0.206 (0.122)
6	0.050 (0.197)	0.039 (−0.271)	0.016 (0.004)	0.044 (0.260)	0.155 (0.079)	0.216 (0.123)
7	0.079 (0.313)	0.043 (−0.386)	0.023 (−0.084)	0.055 (0.167)	0.241 (0.136)	0.251 (0.132)
8	0.059 (0.257)	0.031 (−0.341)	0.014 (−0.076)	0.033 (0.076)	0.206 (0.133)	0.235 (0.138)
9	0.035 (0.143)	0.017 (−0.277)	0.011 (−0.074)	0.026 (0.019)	0.087 (0.085)	0.115 (0.118)
10	0.021 (−0.211)	0.035 (−0.242)	0.016 (−0.009)	0.033 (0.175)	0.075 (0.044)	0.162 (0.084)
11	0.026 (−0.075)	0.030 (−0.231)	0.015 (−0.025)	0.042 (0.244)	0.093 (0.056)	0.164 (0.097)
12	0.016 (−0.237)	0.018 (−0.138)	0.017 (0.038)	0.023 (0.086)	0.066 (0.034)	0.084 (0.040)
13	0.076 (0.324)	0.051 (−0.586)	0.019 (−0.051)	0.021 (−0.145)	0.290 (0.155)	0.274 (0.132)
14	0.074 (0.351)	0.032 (−0.446)	0.007 (−0.066)	0.015 (−0.146)	0.331 (0.187)	0.214 (0.108)
15	0.072 (0.337)	0.036 (−0.408)	0.026 (−0.054)	0.031 (0.021)	0.299 (0.156)	0.231 (0.096)
16	0.054 (0.229)	0.038 (−0.462)	0.019 (0.013)	0.018 (−0.147)	0.239 (0.136)	0.211 (0.096)
17	0.042 (0.174)	0.026 (−0.315)	0.004 (−0.004)	0.044 (0.176)	0.209 (0.139)	0.202 (0.111)
18	0.037 (0.156)	0.018 (−0.076)	0.008 (0.107)	0.038 (0.205)	0.193 (0.132)	0.157 (0.098)

Based on these results, we have analyzed the bond-condensed Fukui functions between atom 1 and the indicated atom and the atom-condensed Fukui function for atom 1 for all molecules in the test set, by using weights from the neutral systems. Table 5 shows that compounds 2–12 have condensed values for f^+_A on atom 1 larger than their respective f^-_1. Hence, these atoms will preferably undergo nucleophilic attack. This behavior is opposite to the rest of the systems, where electrophilic attack on these atoms is preferable. A similar performance is observed in the previous results[45] calculated by using the Mulliken approach, which are included in the table to facilitate comparison. The condensed values using either the Hirshfeld-I AIM or a Mulliken approach show the same tendencies in reactivity. However, the correlation in the values is not as good, especially for f^+_A where the correlation coefficient R^2 between the Mulliken and Hirshfeld-I approaches is around 0.57, as opposed to f^-_A where R^2 equals 0.92. The correlation coefficients between both approaches for f_{AB} are 0.52 and 0.82 for f^+_{12} and f^-_{12}, respectively. Table 6 reports the counterpart results to those calculated by using the Hirshfeld-I approach shown in Table 5 arising from the RMF approach. To compare the behavior of FMR and RMF procedures, we have also calculated the correlation coefficients between the FMR and RMF series of results. In general, the condensed values show similar reactivity tendencies for both FMR and RMF approaches. However, again the correlation in the values is rather weak, for example, for the f^+_A quantities the correlation coefficient R^2 between both approaches is around 0.36 whereas for f^-_A ones R^2

Table 6. B3LYP 6-31G* bond Fukui functions between the indicated atoms and the atom-condensed Fukui function on atom 1 for addition or removal of an electron for each molecule by using the RMF approach.

System	f^-_{12}	f^+_{12}	f^-_{23}	f^+_{23}	f^-_1	f^+_1
1	0.187	0.126	0.068	0.082	0.333	0.293
2	0.061	0.057	−0.023	0.018	0.173	0.258
3	0.044	0.060	−0.016	0.044	0.151	0.290
4	0.108	0.050	0.028	0.049	0.158	0.307
5	0.031	0.056	−0.001	0.058	0.132	0.277
6	0.079	0.072	0.000	0.054	0.151	0.263
7	0.123	0.095	0.024	0.057	0.270	0.269
8	0.098	0.091	0.013	0.044	0.244	0.241
9	0.064	0.081	0.015	0.037	0.141	0.221
10	0.032	0.072	0.005	0.039	0.062	0.174
11	0.035	0.066	−0.003	0.054	0.137	0.316
12	0.021	0.026	0.010	0.025	0.045	0.066
13	0.104	0.088	0.027	0.039	0.396	0.272
14	0.104	0.065	0.002	0.035	0.456	0.194
15	0.096	0.070	0.019	0.000	0.421	0.210
16	0.072	0.065	0.038	0.043	0.367	0.212
17	0.064	0.055	−0.025	0.045	0.310	0.224
18	0.050	0.010	−0.012	0.041	0.290	0.221

equals 0.89. For the f_{AB} quantities, the correlation coefficients turn out to be 0.55 and 0.97 for f^+_{12} and f^-_{12}, respectively. As a consequence, one should restrict oneself to the study of reactivity tendencies and not attempt any numeric comparison. This is, however, a universal observation whenever AIM dependent quantities are compared.

5. Conclusions

In this work, a new partitioning of the Fukui matrix has been proposed within the Hirshfeld-I framework. Starting from the molecular Fukui matrix and by using a symmetrized product of weight functions for the atoms in a molecule, one- and two-atom Fukui matrices have been obtained, giving rise to atom- and bond-condensed matrices, respectively. The resulting matrices $\mathbf{F}_{AB}$ are then expressed in terms of the molecular orbitals and diagonalized, what leads to a set of Fukui orbitals for each isolated atom A and each pair of atoms AB in the molecule. The trace of the $\mathbf{F}_{AB}$ matrices corresponds to a f_{AB} Fukui function. The more traditional atom-condensed Fukui functions can straightforwardly be recovered from the f_{AB} set.

For all the studied molecules, it has been found that the condensed Fukui functions do not strongly depend on the source of the weight functions (neutral molecule or ionic species) used in the computation. Based on the results presented in molecules of a series of ethylene derivatives, the obtained Fukui matrices and functions turn out to be useful tools to attach a degree of reactivity to atoms and chemical bonds in a molecule, and to identify the molecular regions preferred in chemical reactions involving electrophilic or nucleophilic attacks. The comparison between the present Hirshfeld-I based Fukui matrix partitioning and the previously reported Mulliken based one leads to similar conclusions for both approaches.

Supporting Information: Results corresponding to the compounds included in Figure 1, except those of the ethylene molecule, which are reported in the text.

Acknowledgments

D.R.A. and P.B. acknowledge the Ministerio de Ciencia, Tecnología e Innovación Productiva (Argentina) and the FWO Vlaanderen (Belgium) for a collaborative research grant VS.0001.14N. D.R.A. acknowledges the Universidad de Buenos Aires (Argentina) and the Consejo Nacional de Investigaciones Científicas y Técnicas (Argentina) for research Grant Nos. UBACYT 20020100100197, PIP 11220090100061, and PIP 11220130100377CO. A.T. and L.L. acknowledge the Universidad del País Vasco (Spain) for research Grant No. UFI11/07. O.B.O. acknowledges the Consejo Nacional de Investigaciones Científicas y Técnicas (Argentina) for research Grant Nos. 11220090100369 and 11220130100311CO. D.V.N. and P.B. acknowledge support from the Research Foundation Flanders (FWO Vlaanderen). The computational resources and services used in this work were provided by the Universidad de Buenos Aires, the Universidad del País Vasco, and the Stevin Supercomputer Infrastructure, provided by the VSC (Flemish Supercomputer Center), funded by Ghent University, the Hercules Foundation, and the Flemish Government—department EWI. P.B. and D.V.N. are members of the QCMM alliance Ghent–Brussels.

Keywords: atoms in molecules · frontier molecular orbitals · Fukui functions

[1] R. Parr, W. Yang, *Density Functional Theory of Atoms and Molecules*, Oxford Science, Oxford, **1989**.
[2] P. Geerlings, F. D. Proft, W. Langenaeker, *Chem. Rev.* **2003**, *103*, 1793–1874.
[3] K. Fukui, H. Shingu, *J. Chem. Phys.* **1952**, *20*, 722–725.
[4] K. Fukui, *Science* **1982**, *218*, 747–754.
[5] W. Yang, R. G. Parr, R. Pucci, *J. Chem. Phys.* **1984**, *81*, 2862–2863.
[6] R. G. Parr, W. Yang, *J. Am. Chem. Soc.* **1984**, *106*, 4049–4050.
[7] P. Ayers, M. Levy, *Theor. Chem. Acc.* **2000**, *103*, 353–360.
[8] C. Cárdenas, P. W. Ayers, A. Cedillo, *J. Chem. Phys.* **2011**, *134*, 174103.
[9] P. Bultinck, C. Cardenas, P. Fuentealba, P. Johnson, P. W. Ayers, *J. Chem. Theory Comput.* **2013**, *9*, 4779–4788.
[10] P. Bultinck, C. Cardenas, P. Fuentealba, P. Johnson, P. W. Ayers, *J. Chem. Theory Comput.* **2014**, *10*, 202–210.
[11] J. P. Perdew, R. G. Parr, M. Levy, J. L. Balduz, *Phys. Rev. Lett.* **1982**, *49*, 1691–1694.
[12] W. Yang, Y. Zhang, P. W. Ayers, *Phys. Rev. Lett.* **2000**, *84*, 5172–5175.
[13] P. Ayers, *J. Math. Chem.* **2008**, *43*, 285–303.
[14] P. Bultinck, D. Clarisse, P. W. Ayers, R. Carbó-Dorca, *Phys. Chem. Chem. Phys.* **2011**, *13*, 6110–6115.
[15] P. Bultinck, D. Van Neck, G. Acke, P. W. Ayers, *Phys. Chem. Chem. Phys.* **2012**, *14*, 2408–2416.
[16] D. R. Alcoba, W. Tiznado, O. B. Oña, A. Torre, L. Lain, *Chem. Phys. Lett.* **2012**, *533*, 114–117.
[17] D. R. Alcoba, L. Lain, A. Torre, O. B. Oña, W. Tiznado, *Chem. Phys. Lett.* **2012**, *549*, 103–107.
[18] D. R. Alcoba, L. Lain, A. Torre, O. B. Oña, E. Chamorro, *Phys. Chem. Chem. Phys.* **2013**, *15*, 9594–9604.
[19] S. Van Damme, P. Bultinck, *J. Mol. Struct. THEOCHEM* **2010**, *943*, 83–89.
[20] S. Van Damme, P. Bultinck, *J. Comput. Chem.* **2009**, *30*, 1749–1757.
[21] W. Yang, W. J. Mortier, *J. Am. Chem. Soc.* **1986**, *108*, 5708–5711.
[22] P. Bultinck, S. Fias, C. Van Alsenoy, P. W. Ayers, R. Carbó-Dorca, *J. Chem. Phys.* **2007**, *127*, 034102.
[23] R. F. W. Bader, *Chem. Rev.* **1991**, *91*, 893–928.

[24] R. F. W. Bader, *Atoms in Molecules, A Quantum Theory,* Oxford Science, Oxford, **1990**.
[25] F. Hirshfeld, *Theor. Chim. Acta* **1977**, *44*, 129–138.
[26] P. Bultinck, C. Van Alsenoy, P. W. Ayers, R. Carbó-Dorca, *J. Chem. Phys.* **2007**, *126*, 144111.
[27] N. Otero, M. Mandado, R. A. Mosquera, *J. Chem. Phys.* **2007**, *126*, 234108.
[28] J. Cioslowski, M. Martinov, S. Mixon, *J. Phys. Chem.* **1993**, *97*, 10948–10951.
[29] A. Morgenstern, C. Morgenstern, J. Miorelli, T. Wilson, M. Eberhart, *Phys. Chem. Chem. Phys.* **2016**, *18*, 5638–5646.
[30] P. Ayers, R. Morrison, R. Roy, *J. Chem. Phys.* **2002**, *116*, 8731–8744.
[31] D. Vanfleteren, D. Van Neck, P. Bultinck, P. W. Ayers, M. Waroquier, *J. Chem. Phys.* **2010**, *132*, 164111.
[32] X. Fradera, M. Solá, *J. Comput. Chem.* **2004**, *25*, 439–446.
[33] A. Aizman, R. Contreras, *J. Chil. Chem. Soc.* **2004**, *49*, 107–111.
[34] M. Gonzalez-Suarez, A. Aizman, R. Contreras, *Theor. Chem. Acc.* **2010**, *126*, 45–54.
[35] E. Matito, M. V. Putz, *J. Phys. Chem. A* **2011**, *115*, 12459–12462.
[36] N. Otero, M. Mandado, *J. Comput. Chem.* **2012**, *33*, 1240–1251.
[37] R. Roy, S. Pal, K. Hirao, *J. Chem. Phys.* **1999**, *110*, 8236–8245.
[38] R. Roy, K. Hirao, S. Pal, *J. Chem. Phys.* **2000**, *113*, 1372–1379.
[39] P. Bultinck, R. Carbó-Dorca, *J. Math. Chem.* **2003**, *34*, 67–74.
[40] P. Bultinck, R. Carbó-Dorca, W. Langenaeker, *J. Chem. Phys.* **2003**, *118*, 4349–4356.
[41] P. Senet, M. Yang, *J. Chem. Sci.* **2005**, *117*, 411–418.
[42] P. W. Ayers, *Phys. Chem. Chem. Phys.* **2006**, *8*, 3387–3390.
[43] J. Melin, P. W. Ayers, J. V. Ortiz, *J. Phys. Chem. A* **2007**, *111*, 10017–10019.
[44] M. Gonzalez-Suarez, A. Aizman, J. Soto-Delgado, R. Contreras, *J. Org. Chem.* **2012**, *77*, 90–95.
[45] P. Bultinck, S. Van Damme, A. Cedillo, *J. Comput. Chem.* **2013**, *34*, 2421–2429.
[46] P. Bultinck, P. W. Ayers, S. Fias, K. Tiels, C. V. Alsenoy, *Chem. Phys. Lett.* **2007**, *444*, 205–208.
[47] S. Van Damme, P. Bultinck, S. Fias, *J. Chem. Theory Comput.* **2009**, *5*, 334–340.
[48] T. Verstraelen, E. Pauwels, F. De Proft, V. Van Speybroeck, P. Geerlings, M. Waroquier, *J. Chem. Theory Comput.* **2012**, *8*, 661–676.
[49] T. Verstraelen, V. Van Speybroeck, M. Waroquier, *J. Chem. Phys.* **2009**, *131*, 044127.
[50] T. Verstraelen, P. W. Ayers, V. Van Speybroeck, M. Waroquier, *Chem. Phys. Lett.* **2012**, *545*, 138–143.
[51] T. Verstraelen, P. Bultinck, V. Van Speybroeck, P. W. Ayers, D. Van Neck, M. Waroquier, *J. Chem. Theory Comput.* **2011**, *7*, 1750–1764.
[52] P. Bultinck, W. Langenaeker, P. Lahorte, F. De Proft, P. Geerlings, C. Van Alsenoy, J. Tollenaere, *J. Phys. Chem. A* **2002**, *106*, 7895–7901.
[53] Gaussian 03 (Revision E.01), M. J. Frisch, G. W. Trucks, H. B. Schlegel, G. E. Scuseria, M. A. Robb, J. R. Cheeseman, J. A. Montgomery, Jr., T. Vreven, K. N. Kudin, J. C. Burant, J. M. Millam, S. S. Iyengar, J. Tomasi, V. Barone, B. Mennucci, M. Cossi, G. Scalmani, N. Rega, G. A. Petersson, H. Nakatsuji, M. Hada, M. Ehara, K. Toyota, R. Fukuda, J. Hasegawa, M. Ishida, T. Nakajima, Y. Honda, O. Kitao, H. Nakai, M. Klene, X. Li, J. E. Knox, H. P. Hratchian, J. B. Cross, V. Bakken, C. Adamo, J. Jaramillo, R. Gomperts, R. E. Stratmann, O. Yazyev, A. J. Austin, R. Cammi, C. Pomelli, J. W. Ochterski, P. Y. Ayala, K. Morokuma, G. A. Voth, P. Salvador, J. J. Dannenberg, V. G. Zakrzewski, S. Dapprich, A. D. Daniels, M. C. Strain, O. Farkas, D. K. Malick, A. D. Rabuck, K. Raghavachari, J. B. Foresman, J. V. Ortiz, Q. Cui, A. G. Baboul, S. Clifford, J. Cioslowski, B. B. Stefanov, G. Liu, A. Liashenko, P. Piskorz, I. Komaromi, R. L. Martin, D. J. Fox, T. Keith, M. A. Al-Laham, C. Y. Peng, A. Nanayakkara, M. Challacombe, P. M. W. Gill, B. Johnson, W. Chen, M. W. Wong, C. Gonzalez, J. A. Pople, Gaussian, Inc., Wallingford, CT, **2004**.
[54] R. Dennington, T. Keith, J. Millam, *GaussView Version 5*, Semichem Inc. Shawnee Mission KS **2009**.

Manuscript received: April 29, 2016
Final Article published: July 6, 2016